Unit 8

Exponential & Logarithmic Functions

Exponential Growth and Decay

Sections 8.1 and 8.2

Exponential Growth

- y-int: $(0, a)$
- HA: $y = 0$
- D: $(-\infty, \infty)$
- R: $(0, \infty)$ if $a > 0$
  R: $(-\infty, 0)$ if $a < 0$

$$f(x) = a \cdot b^x \ ; \ where \ a > 0, b > 1$$

positive

To – Do:

1. Graph $y = \frac{1}{3} \cdot 2^x$, and $y = 3 \cdot 2^x$. Compare with the graph of $y = 2^x$

   Parent Graph

2. Graph $y = -\frac{1}{5} \cdot 2^x$, and $y = -5 \cdot 2^x$. Compare with the graph of $y = 2^x$

3. Describe the effect of a on the graph of $y = a \cdot 2^x$ when a is positive and when a is negative.

Graph the functions

- List Parent Graph
- List Transformations
- Plot 2 points
- List/draw HA
- State D and R

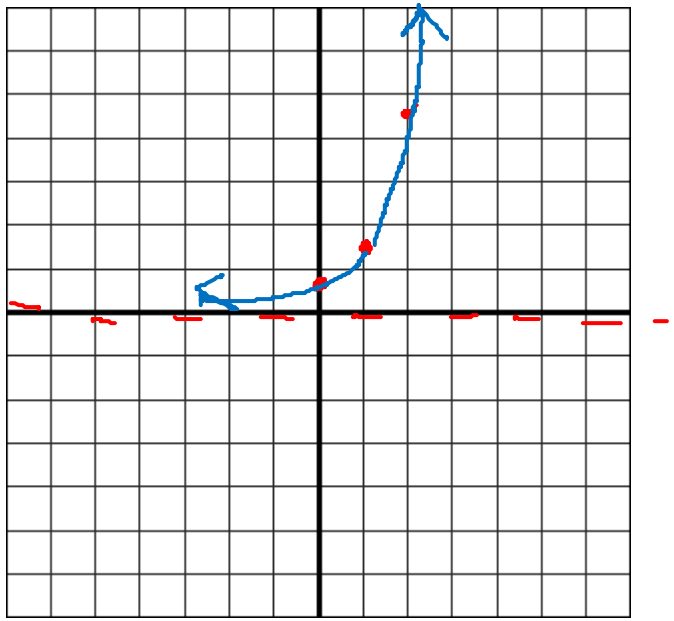
- $y = \frac{1}{2} \cdot 3^x$

P.G.: $y = 3^x$

TR: Vertical Shrink

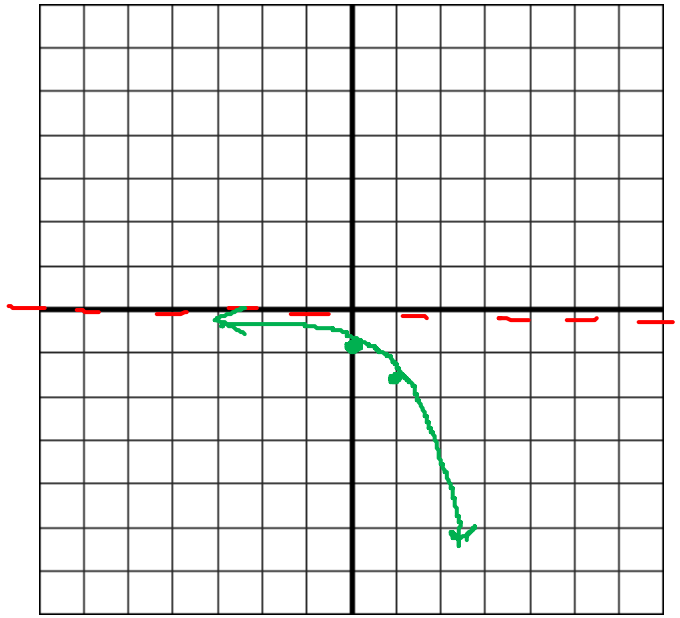
Pts: $(0, 1/2)$ $(1, \frac{3}{2})$

- HA: $y = 0$
- D: $(-\infty, \infty)$
- R: $(0, \infty)$



- $y = -\left(\frac{3}{2}\right)^x$

- PG: $y = \frac{3}{2}^x$
- TR: Reflect x-axis
- Pts: $(0, -1)$ $(1, -3/2)$
- HA: $y = 0$
- D: $(-\infty, \infty)$
- R: $(-\infty, 0)$



Graph the functions

- List Parent Graph
- List Transformations
- Plot 2 points
- List/draw HA
- State D and R

- $y = 3 \cdot 2^{x-1} - 4$

P.G.: $y = 2^x$

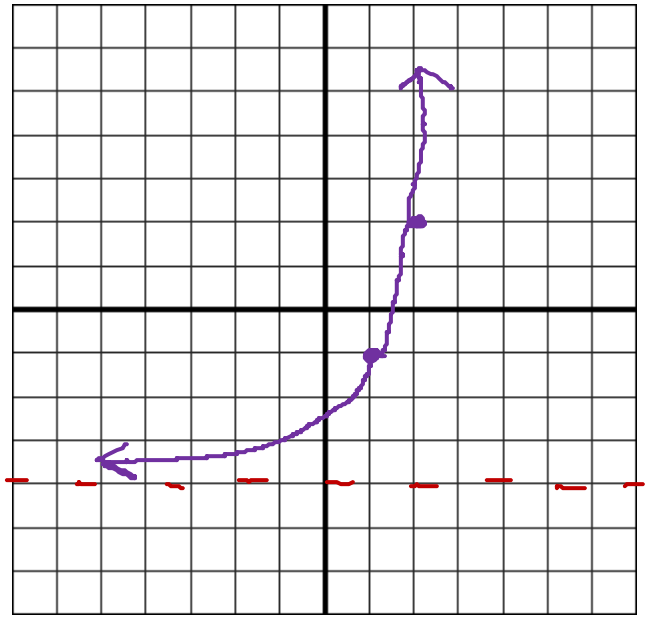
TR: Vert. stretch
Right 1
Down 4

Pts: (1, −1)
(2, 2)

HA: $y = -4$

D: $(-\infty, \infty)$

R: $(-4, \infty)$



- $y = -3^x - 2$

P.G.: $y = 3^x$

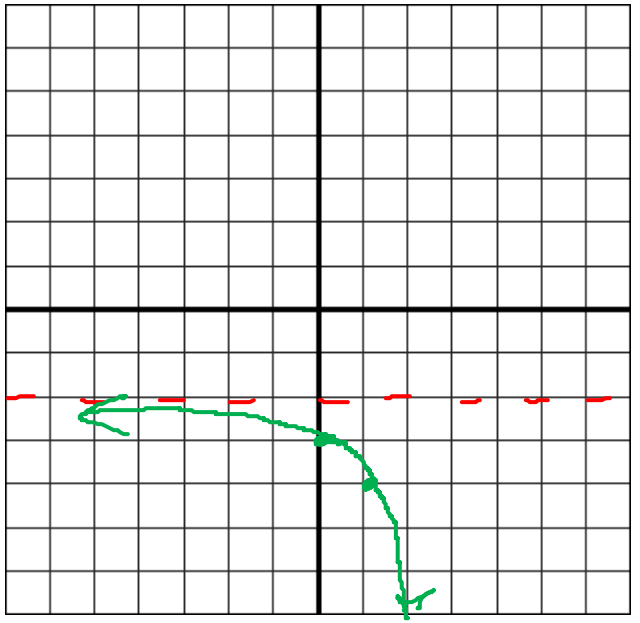
TR: Reflect x-axis
Down 2

Pts: (0, −3)
(1, −5)

HA: $y = -2$

D: $(-\infty, \infty)$

R: $(-\infty, -2)$



Using Exponential Growth Models

$h = 1.313(1 + .98)^t$

$h = 1.313(1.98)^t$ (1.98 = growth factor)

$$y = a(1 + r)^t$$

a = initial amount
r = rate (% ⇒ decimal)
t = time

In January, 1993, there were about 1,313,000 internet hosts. During the next 5 years, the number of hosts increased by 98% per year. (1,313,000 = 1.313 million)

a) Write a model giving the number h (in millions) of hosts t years after 1993. About how many hosts were there in 1996?

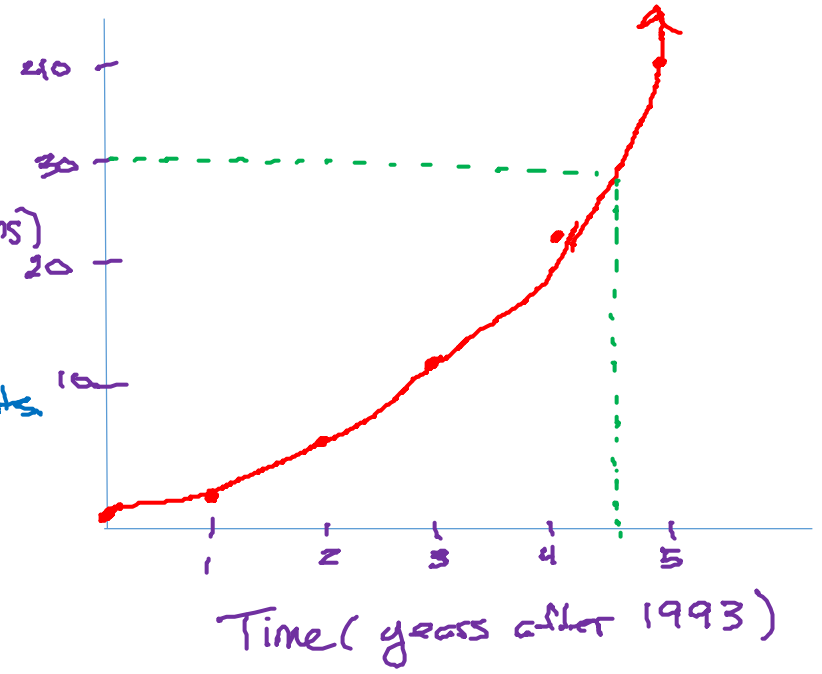
b) Graph the model

c) In what year were there 30 million hosts?

$h = 1.313(1.98)^3$

$h = 10.192 \approx 10{,}192{,}020$ hosts

1997



(0, 1.313) (1, 2.6) (2, 5.1) (3, 10.192) (4, 20.2) (5, 40)

Compound Interest

A = Balance (ending amount)

P = Principle (starting am't)

r = rate (% ⇒ dec.)

n = # of times interest is compounded per year

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

t = total investment period

You deposit $1000 in an account that pays 8% annual interest. Find the balance after 1 year if the interest is compounded with the given frequency.

a.) annually $n = 1$

$A = 1000\left(1 + \frac{.08}{1}\right)^{1 \cdot 1}$

$A = \$1080.00$

b.) quarterly $n = 4$

$A = 1000\left(1 + \frac{.08}{4}\right)^{4 \cdot 1}$

$A \approx \$1082.43$

c.) daily $n = 365$

$A = 1000\left(1 + \frac{.08}{365}\right)^{365 \cdot 1}$

$A \approx \$1083.27$

Last Question…

$a = 100$

$y = 100(1.3)^{10}$

$y = 1378.....$

$\frac{1378}{100} = 13.78$

If the population of a town increased by 30% per year over a period of 10 years, by how many times did the population increase in the ten-year period?

13.8 times bigger

$y = a(1 + r)^t$

$y = (1.3)^{10}$

$y = 13.78.....$

Exponential Decay

$$f(x) = a \cdot b^x \ ; \ where \ a > 0 \ and \ 0 < b < 1$$

pos.

fraction btw 0 & 1

State whether the following functions are exponential growth or decay

$f(x) = 5\left(\frac{2}{3}\right)^x$

fraction btw 0 & 1

Decay

$f(x) = 8\left(\frac{3}{2}\right)^x$

> 1

Growth

$f(x) = 10(3)^{-x}$

$= 10\left(\frac{1}{3}\right)^x$

Decay

$f(x) = 10\left(\frac{1}{3}\right)^{-x}$

$= 10(3)^x \Rightarrow$ Growth

Graph the functions

- List Parent Graph
- List Transformations
- Plot 2 points
- List/draw HA
- State D and R

- $y = 3 \cdot 4^{-x}$

Decay: $3 \cdot \left(\frac{1}{4}\right)^x$

P.G: $y = \frac{1}{4}^x$ $y = 4^{-x}$

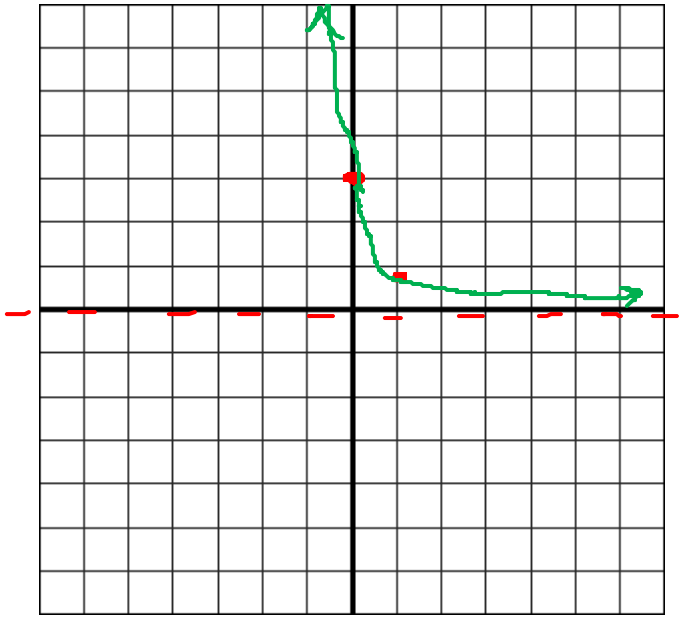
TR: Vert. Stretch

Pts: (0, 3) (1, 3/4) (-1, 12)

HA: $y = 0$

D: $(-\infty, \infty)$

R: $(0, \infty)$



- $y = -5\left(\frac{2}{3}\right)^x$

P.G.: $y = (2/3)^x$

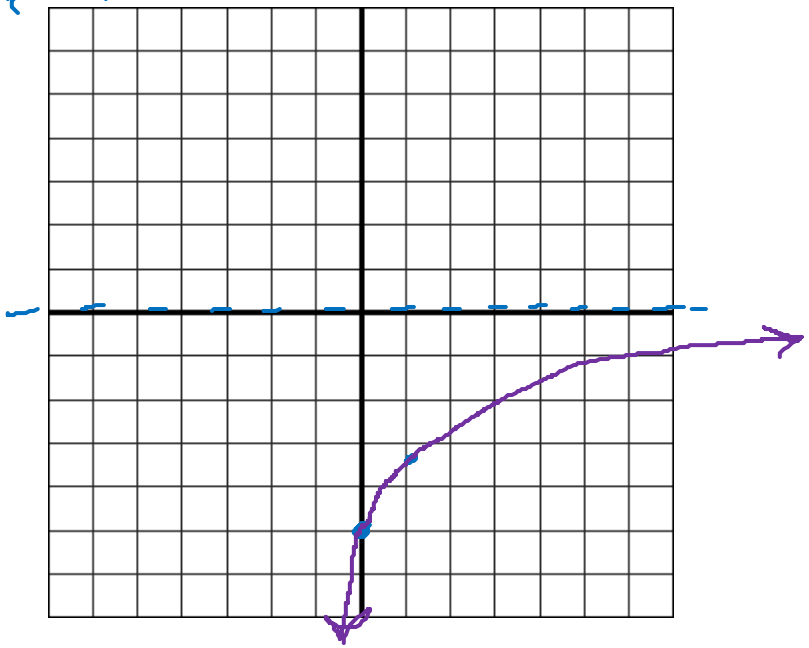
TR: Reflect x-axis
Vert. stretch

Pts: (0, -5)
(1, -10/3)
(1, -3 1/3)

HA: $y = 0$

D: $(-\infty, \infty)$

R: $(-\infty, 0)$



Graph the functions

- List Parent Graph
- List Transformations
- Plot 2 points
- List/draw HA
- State D and R

- $y = -3\left(\frac{1}{2}\right)^{x+2} + 1$

P.G. $y = \left(\frac{1}{2}\right)^x$

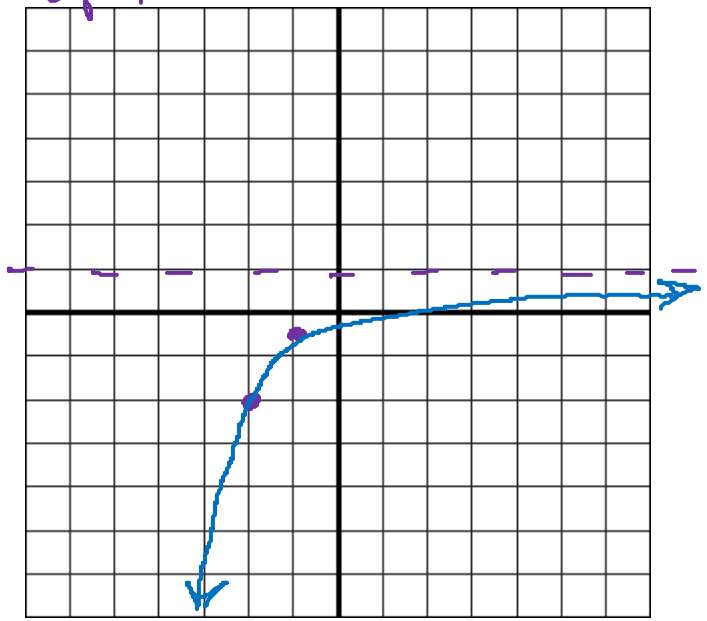
Pts: $(-2, -2)$
$(-1, -\frac{1}{2})$

TR: Reflect x-axis
V. Stretch
Left 2
Up 1

HA: $y = 1$

D: $(-\infty, \infty)$

R: $(-\infty, 1)$



- $y = 5 \cdot 8^{-(x+1)} - 2$

$y = 5 \cdot \left(\frac{1}{8}\right)^{(x+1)} - 2$

P.G.: $y = \left(\frac{1}{8}\right)^x$ $y = 8^{-x}$

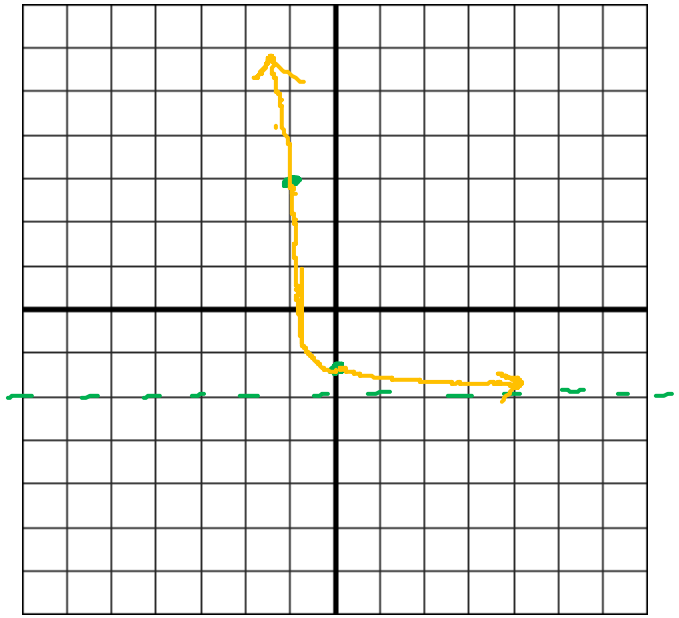
TR: V. Stretch
Left 1
Down 2

Pts: $(-1, 3)$
$(0, -1\frac{3}{8})$
$= \frac{5}{8} - 2$
$= \frac{5}{8} - \frac{16}{8} = \frac{-11}{8}$
$= -1\frac{3}{8}$

HA: $y = -2$

D: $(-\infty, \infty)$

R: $(-2, \infty)$



Using Exponential Decay Models

$$y = a(1 - r)^t$$

You buy a new car for $24,000. The value *y* of the car decreases by 16% each year.

a) Write an exponential decay model for the value of the car. Use the model to estimate the value after 2 years.

b) Graph the model

c) Use the graph to estimate when the car will have a value of $12,000

Textbook Practice Problems

Section 8.1

- Pages 469-470
- #13-24, 29, 31, 34, 41, 49-54

Section 8.2

- Pages 477-478
- #~~11-24~~, 25, 31, 36, 40, 43-56

HW

Apps